

Paper Reference 9FM0/3D
Pearson Edexcel
Level 3 GCE

Further Mathematics
Advanced
PAPER 3D: Decision Mathematics 1

Friday 23 June 2023 – Afternoon

Time: 1 hour 30 minutes

YOU MUST HAVE
Mathematical Formulae and Statistical Tables (Green),
calculator

YOU WILL BE GIVEN
Diagram Booklet
Answer Booklet

X72799A

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

INSTRUCTIONS

In the boxes on the Answer Booklet and on the Diagram Booklet, write your name, centre number and candidate number.

Do not return the Question Paper with the Answer Booklet.

Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.

Answer the questions in the Answer Booklet or on the separate diagrams in the Diagram Booklet – there may be more space than you need.

Do NOT write on this Question Paper.

You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

Inexact answers should be given to three significant figures unless otherwise stated.

Turn over

INFORMATION

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

There are **7** questions in this Question Paper. The total mark for this paper is **75**.

The marks for EACH question are shown in brackets – use this as a guide as to how much time to spend on each question.

There are two copies of each diagram in case you need them.

ADVICE

Read each question carefully before you start to answer it.

Try to answer every question.

Check your answers if you have time at the end.

1. Refer to Diagram 1, Diagram 2 and Diagram 3 in the Diagram Booklet.

Diagram 1 shows the graph **G**

- (a) State whether **G** is Eulerian, semi-Eulerian, or neither, giving a reason for your answer.
(1 mark)
- (b) Write down an example of a Hamiltonian cycle on **G**
(1 mark)
- (c) State whether or not **G** is planar, justifying your answer.
(1 mark)
- (d) State the number of arcs that would need to be added to **G** to make the graph K_5
(1 mark)

(continued on the next page)

1. continued.

Direct roads between five villages, **A, B, C, D** and **E**, are represented in Diagram 2.

The weight on each arc is the time, in minutes, required to travel along the corresponding road.

Floyd's algorithm is to be used to find the complete network of shortest times between the five villages.

(e) For the network represented in Diagram 2, complete the initial time matrix in the Answer Booklet.

(1 mark)

The time matrix after four iterations of Floyd's algorithm is shown in Diagram 3 in the Diagram Booklet.

(f) In the Answer Booklet, perform the final iteration of Floyd's algorithm that follows from Diagram 3, showing the time matrix for this iteration.

(2 marks)

(Total for Question 1 is 7 marks)

- 2. Refer to Diagram 4, Diagram 5, Diagram 6 and Diagram 7 in the Diagram Booklet.**

The network in Diagram 4 shows the activities that need to be undertaken to complete a project.

Each activity is represented by an arc and the duration, in hours, of the corresponding activity is shown in brackets.

- (a) (i) Complete Diagram 5 in the Diagram Booklet to show the early event times and the late event times.**

- (ii) State the minimum completion time of the project.**

(3 marks)

(continued on the next page)

2. continued.

Diagram 6 in the Diagram Booklet lists the number of workers required for each activity in the project.

Each worker is able to do any of the activities.

Once an activity is started it must be completed without interruption.

It is given that each activity begins at its earliest possible start time.

(b) (i) On Diagram 7 in the Diagram Booklet, draw a resource histogram to show the number of workers required at each time.

(ii) Hence state the time interval(s) when six workers are required.

(4 marks)

(Total for Question 2 is 7 marks)

3. Refer to Diagram 8 and Diagram 9 in the Diagram Booklet.

Diagram 8 represents a network with nodes, **A, B, C, D, E, F, G, H** and **J**

The number on each edge gives the length of the corresponding edge.

- (a) (i) Use Dijkstra's algorithm and Diagram 9 in the Diagram Booklet to find the shortest path from **A** to **J**
- (ii) State the length of the shortest path from **A** to **J**

(6 marks)

(continued on the next page)

3. continued.

One application of Dijkstra's algorithm has order n^2 , where n is the number of nodes in the network.

It takes a computer **0.0312** seconds to find the shortest path from a given start node to a given end node in a network of **9** nodes.

- (b) Calculate approximately how long it would take, in minutes, for the computer to find the shortest path from a given start node to a given end node for a network of **9000** nodes.
(2 marks)

(Total for Question 3 is 8 marks)

4. The eleven distinct numbers listed below are to be packed into bins of size 40

15	22	3	9	23	x
5	4	18	20	13	

It is known that x

- is an integer less than 40
- is the largest number in the list

- (a) Explain why it IS NOT possible to pack the numbers into 3 bins of size 40
(1 mark)

Given that it IS possible to pack the numbers into 4 bins of size 40

- (b) determine the range of values for x
(2 marks)

(continued on the next page)

4. continued.

**(c) Use the first–fit bin packing algorithm to determine how the numbers can be packed into bins of size 40
(3 marks)**

**(d) Carry out a quick sort to produce a list of the numbers in descending order.
You should show the result of each pass and identify your pivots clearly.
(4 marks)**

When the first–fit decreasing bin packing algorithm is applied to the list, neither the 15 nor the 13 is placed in the first bin.

**(e) Determine the value of x
You must give reasons for your answer.
(2 marks)**

(Total for Question 4 is 12 marks)

5. Refer to Diagram 10 and Diagram 11 in the Diagram Booklet.

[The total weight of the network is 423]

Direct roads between nine towns, **A, B, C, D, E, F, G, H** and **J**, are represented in Diagram 10.

The number on each arc represents the length, in miles, of the corresponding road.

Diagram 11 in the Diagram Booklet shows the shortest distances, in miles, between the nine towns.

A route is needed that minimises the total distance required to traverse each road at least once.

The route must start at **F** and finish at **J**

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5. continued.

(a) (i) By considering the pairings of all relevant nodes, find the roads that would need to be traversed twice.

(ii) State the total length of this route.

(5 marks)

(b) Starting at A, use Prim's algorithm to find the minimum spanning tree for the table of shortest distances.

You must state the order in which you select the arcs of your tree.

(3 marks)

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5. continued.

Pete needs to visit all nine towns, starting and finishing in the same town, and wishes to minimise the total distance he travels.

(c) Starting at **G, use the nearest neighbour algorithm on the table of shortest distances to find an upper bound for the length of Pete's route.**

Write down the route that gives this upper bound.

(2 marks)

(d) By deleting **G and all of its arcs, find a lower bound for the length of Pete's route.**

(2 marks)

(continued on the next page)

5. continued.

Pete decides to take the route he found in (c).

(e) Interpret the route in terms of the actual towns visited.

(1 mark)

(Total for Question 5 is 13 marks)

6. Refer to Diagram 12 and Diagram 13 in the Diagram Booklet.

Diagram 12 shows a precedence table with the twelve activities required to complete a project.

- (a) In the space provided in the Answer Booklet, draw the activity network described in the precedence table, using activity on arc. Your activity network must contain the minimum number of dummies only.
(5 marks)

(continued on the next page)

6. continued.

Diagram 13 shows a partially completed cascade chart for the project.

The non-critical activities F, J and K are NOT shown in Diagram 13.

The time taken to complete each activity is given in hours and the project is to be completed in the minimum possible time.

(b) State the critical activities.

(1 mark)

Given that the total float of activity F is 2 hours,

(c) state the duration of activity F

(1 mark)

(continued on the next page)

6. continued.

The duration of activity **J** is **x** hours, and the duration of activity **K** is **y** hours, where **$x > 0$** and **$y > 0$**

(d) (i) State, in terms of y , the maximum possible total float for activity **K**

(ii) State, in terms of x and y , the total float for activity **J**

(2 marks)

(Total for Question 6 is 9 marks)

7. Refer to Diagram 14 and Diagram 15 in the Diagram Booklet.

A publisher plans to produce three versions of the same book: a paperback, a hardcover, and a deluxe edition.

- Each paperback takes **4** minutes to print and **1** minute to bind
- Each hardcover takes **8** minutes to print and **5** minutes to bind
- Each deluxe edition takes **15** minutes to print and **12** minutes to bind

The printing machine is available for at most **150** hours and the binding machine must be used for at least **60** hours.

The publisher decides to produce

- at least **1600** books in total
- at least three times as many paperbacks as hardcovers

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7. continued.

The profit on each paperback sold is £8, the profit on each hardcover sold is £20 and the profit on each deluxe edition sold is £40

Let x , y and z represent the number of paperbacks, hardcovers and deluxe editions produced.

- (a) Formulate this as a linear programming problem, stating the objective and listing the constraints as simplified inequalities with integer coefficients.**
- (5 marks)**

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7. continued.

The publisher decides to solve this linear programming problem by using the two–stage simplex method.

(b) Set up an initial tableau in the Answer Booklet for solving this problem using the two–stage simplex method.

As part of your solution, you must show how

- the constraints have been made into equations by using slack variables, exactly two surplus variables and exactly two artificial variables**
- the rows for the two objective functions are formed**

(6 marks)

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7. continued.

After two iterations of the first stage of the two-stage simplex method, the tableau shown in Diagram 14 is obtained.

- (c) Taking the most negative number in the profit row to indicate the pivot column, perform one complete iteration of the second stage of the two-stage simplex method to obtain a new tableau using the table in the Answer Booklet. Make your method clear by stating the row operations you use.**
- (5 marks)**

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7. continued.

After three iterations of the second stage of the two-stage simplex method, the tableau shown in Diagram 15 is obtained.

Given that the publisher produces the optimal number of each version of the book,

(d) (i) state the maximum profit the publisher can earn,

(ii) find the number of hours the binding machine will be used.

(2 marks)

(e) Give a reason why the publisher may not earn the profit stated in (d)(i).

(1 mark)

(Total for Question 7 is 19 marks)

TOTAL FOR PAPER IS 75 MARKS

END OF PAPER
